Syllabus for MA 59500IDL
Infinite dimensional Lie Algebras and Applications
Spring 2025

1. About the Course

This course is a detailed introduction into the structure and representation theory of some of the most important infinite dimensional Lie algebras: Heisenberg algebras, Kac-Moody algebras, and Virasoro algebra. The course is expected to fit a wide range of students: graduate and advanced undergraduate mathematics students, as well as graduate physics students.

Prerequisites: Basic notions from algebra (especially linear algebra). Familiarity with basic results on simple finite dimensional Lie algebras is welcomed but is not mandatory.

Tentative list of topics:

- Heisenberg algebra, Virasoro algebra, and affine $\widehat{\mathfrak{g}}$ as universal central extensions
- Representations of the Heisenberg algebra, the Virasoro algebra, and affine $\widehat{\mathfrak{sl}}_n$ via Lie algebras $\mathfrak{gl}_\infty, \mathfrak{a}_\infty$, and application to integrable systems
- Boson-fermion correspondence: vertex operator construction and Schur polynomials
- Feigin-Fuchs-Kac determinant formula for Virasoro and the region of unitarity
- The Sugawara construction and the Goddard-Kent-Olive construction
- Structure and representation theory of Kac-Moody algebras
- The Weyl-Kac character formula and the Kac-Macdonald identities
- Shapovalov-Jantzen-Kac-Kazhdan determinant formula for Kac-Moody algebras

2. Lectures

Time: TTh 12:00–1:15pm
Location: MATH 215
Course CRN: #29564
Course Credits: 3

Instructor: Sasha Tsymbaliuk
Email: otsymbal@purdue.edu (emails will be responded within 24h Mon-Fri)
Office hours: TTh 1:20–2:20pm (Math Building 620), 1 flexible hour upon request
Instructional Modality: Face-to-Face (or Online if I need to quarantine or travel)

3. References

The material of this course is based on:

(1) Book "*Bombay lectures on highest weight representations of infinite dimensional Lie algebras*" by V. Kac, A. Raina, N. Rozhkovskaya, Adv. Ser. Math. Phys. 29, World Sci. Publ. Hackensack, NJ (2013), xii+237 pp, ISBN: 978-981-4522-19-9.
(2) Expository paper "*Representations of contragredient Lie algebras and the Kac-Macdonald identities*" by B. Feigin and A. Zelevinsky (1971), 25–77.
(3) Book "*Infinite dimensional Lie algebras*" by V. Kac, 3rd edition, Cambridge University Press, Cambridge (1990), xxii+400 pp, ISBN:0-521-37215-1.

A link to (1) and a copy of (2) are freely available through Brightspace→Content→Literature.

4. Requirements

If you are taking this course for credit, it will be required to solve biweekly homework assignments. The homework will be posted each other Thursday and due two Thursdays afterwards, and will consist of 8–11 problems of various difficulty. There will be no exams in this course.

To get $A-$, a graduate student is expected to solve most of the simpler problems, while to get grades $A, A+$ one should also solve some of the harder (marked by an asterisk *) or more technical problems. The University's policy is that students who get at least 97% of the total points in this course are guaranteed an A+, 93% guarantees an A, 90% an A-, 87% a B+, 83% a B, 80% a B-, 77% a C+, 73% a C, 70% a C-, 67% a D+, 63% a D, and 60% a D-. For each of these grades, the lower percentage will most probably suffice (as noted above).

5. Resources and Collaboration

While solutions are often available online, please make every effort to solve problems yourselves (in case you had to look up for hints or solutions, please cite the source accordingly).

Working in groups is fine, but each person should write up their solutions independently.

6. Attendance

Attendance is expected overall (frequent absences may affect the grade).

Do not come to class if you are feeling ill, but do email me with the subject line: Absence (no need to describe symptoms, but please notify you are feeling ill and cannot come to class).

7. Academic Adjustements for Students with Disabilities

Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247, as soon as possible.

If the Disability Resource Center (DRC) has determined reasonable accommodations that you would like to utilize in this class, you must send your Course Accommodation Letter to the instructor. Instructions on sharing your Course Accommodation Letter can be found

by visiting: https://www.purdue.edu/drc/students/course-accommodation-letter.php. Additionally, you are strongly encouraged to contact the instructor as soon as possible to discuss implementation of your accommodations.

8. Emergency Preparation

In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructors via email.

A link to Purdue's Information on Emergency Preparation and Planning is located on our Brightspace under "University Policies and Statements". This website covers topics such as Severe Weather Guidance, Emergency Plans, and a place to sign up for the Emergency Warning Notification System. I encourage you to download and review the Emergency Preparedness for Classrooms document (PDF) or (Word). The first day of class, I will review the Emergency Preparedness plan for our specific classroom, following Purdue's required Emergency Preparedness Briefing. Please make note of items like:
– The location to where we will proceed after evacuating the building if we hear a fire alarm.
– The location of our Shelter in Place in the event of a tornado warning.
– The location of our Shelter in Place in the event of an active threat such as a shooting.